RESEARCH ARTICLE

A comparative Study of Atomic Entanglement in Single Mode Jaynes Cummings Model with Multiphoton Interaction

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ABSTRACT: In the present paper, standard Jaynes Cummings model involving multiphoton interaction between the field and the atom has been considered. We study the variation of entanglement properties of a pair of two–level Rydberg atoms passing one after another into a lossless cavity with single mode multiphoton interaction. The initial joint state of two successive atoms that enter the cavity is unentangled. Interactions mediated by the cavity field results in the final two atoms mixed entangled type state. The entanglement of formation of the joint two atom state as a function of the Rabi angle g_t is calculated for Fock state field and Coherent state field. A comparative analysis is done to study the extent of atoms being entangled when they interact through a cavity mode of single frequency but via different orders of interaction.

KEYWORDS: Single mode multiphoton Jaynes Cummings Model; Entanglement; Fock State; Coherent State

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1. INTRODUCTION

The Jaynes Cummings model and its generalizations have acquired considerable interest to Physicists with the experimental realisation of the model as well as the non-classical effects predicted by it. Quantum Entanglement is one of the most striking phenomena occurring as an outcome of atom field interaction [1]. It is finding widespread use in quantum information processing, quantum cryptography and quantum teleportation [2,3].

The simplest scheme to investigate the atom-field entanglement is the Jaynes Cummings model [4] that describes the interaction between a two-level atomic system and a single mode quantized radiation field taking into account the near resonance linear coupling. The system of two two-level atoms inside a cavity has been considered as it is experimentally feasible and also it is paradigm to study the evolution of entanglement. In the present work, we have considered the case of multiphoton interaction in a lossless cavity ($Q \to \infty$) of a single mode through which two two-level Rydberg atoms pass one after another. The first atom interacts with the cavity field via two photon process and leaves the cavity. The second atom then enters the cavity and interacts with the field with the changes made by its interaction with the first atom. It has been shown earlier [5,6] that the two atoms get entangled in the process even though they do not interact directly.

The unitary transformation method [7] has been used to solve the multiphoton JCM exactly and then entanglement features for single mode two photon, single mode three photon, and single mode four photon processes are studied.

The quantization of atom – atom entanglement is important for the generation of entangled qubits. The single mode analysis is the simplest mode of interaction between radiation and matter, but it illustrates several non-classical properties of the composite atom field system. In the present study a comparative analysis provided important facts regarding maximally quantized states. The phenomenon of entanglement and superposition of states opens new field of computation and realization of quantum systems.

2. JAYNES CUMMINGS MODEL WITH MULTIPHOTON INTERACTION

The single-mode m photon Jaynes Cummings model is an effective two–level atom interacting with a cavity field of frequency ω viam-photon process.

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The Hamiltonian for the model in the rotating wave approximation is written as [4-6,8].

$$\hat{H} = \hbar \frac{\omega_0}{2} \hat{\sigma}_3 + \hbar\omega \hat{a}^\dagger \hat{a} + \hbar g(\hat{S}_- + \hat{S}_+) \tag{1}$$

where $\hat{S}_-$ *and* $\hat{S}_+$ are given by

$$\hat{S}_- = (\hat{a}^\dagger)^m \hat{\sigma}_- \quad ; \quad \hat{S}_+ = (\hat{a})^m \hat{\sigma}_+ \; (2)$$

Here ω_o is the transition frequency and g is the atom-field coupling constant, $\hat{\sigma}_3$ is the inversion operator and $\hat{\sigma}_+, \hat{\sigma}_-$ are the Pauli raising and lowering operators respectively. $\hat{a}$ and $\hat{a}^\dagger$ are the creation & annihilation operators for the cavity mode. The atomic and field operators involved obey the following commutation relations.

$$[\hat{\sigma}_3, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm, \quad [\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_3, \quad [\hat{a}, \hat{a}^\dagger] = 1 \tag{3}$$

Denoting by $|a\rangle$ and $|b\rangle$ respectively the higher and lower eigenkets of the isolated atom and by $|n\rangle$ the eigen state of the free field with frequency ω,the basis eigenkets of the interacting system can be designated by $|a,n\rangle$ and $|b,n\rangle$.We consider the atoms in their upper states at the start of their individual journey through the cavity. Hence, the initial condition for the system when the first atom enters the field is

$$|\psi(t=0)\rangle = \sum_{n=0}^{\infty} C_n |a_1, n\rangle \tag{4}$$

where $|n\rangle$ represent the cavity photon number states with an initial distribution

$P_n = |C_n|^2 \;(i = 1,2)$. a_1 represents the upper state of the first atom. At resonance $m \; times(\omega) \approx \omega_0$ and the time evolution of the atom-field system wavefunction can be written as

$$|\psi(t)\rangle = \hat{T}(t)\,|\psi(t=0)\rangle = e^{-i\hat{H}t}|\psi(t=0)\rangle \tag{5}$$

In the Heisenberg representation, at two photon resonance in which case $m \; times \; \omega \approx \omega_0$, the time dependence of $\hat{H}$ given by eq. (1) drops out. Using the Hamiltonian of eq. (1), we get

$$\hat{T} = \exp[-it\{\frac{\omega_0}{2}\hat{\sigma}_3 + \omega\, \hat{a}^\dagger \hat{a} + g(\hat{S}_+ + \hat{S}_-)\}] \tag{6}$$

Expanding the expression of $\hat{T}(t)$ given by eqn. (6) and by operating each term on the initial state $|\psi(t=0)\rangle$, at resonance m times $(\omega) \approx \omega_0$, all the terms can be summed up giving us the wavefunction $|\psi_1(t)\rangle$ in a simple form [9-10].

$$|\psi_1(t)\rangle = \cos gt[\sqrt{(n+1)}\sqrt{(n+2)}..\sqrt{(n+m)}]|a_1, n\rangle - i\sin gt[\sqrt{(n+1)}\sqrt{(n+2)}..\sqrt{(n+m)}]|b_1, n+m\rangle \tag{7}$$

where t is now the duration of the atom-field interaction.

The second atom enters the cavity at a time t at which the first atom has already left the cavity. Since $Q=\infty$, the cavity field statistics remain unchanged in the duration between the first atom leaving the cavity and the second atom entering the cavity. Thus, the initial condition for the second atom can be written as $|\psi_2(t=0)\rangle = |a_2\rangle|\psi_1(\tau)\rangle$ so that we get

$$|\psi_2(t=0)\rangle = \cos gt[\sqrt{(n+1)}\sqrt{(n+2)}..\sqrt{(n+m)}]|a_1,a_2,n\rangle - i\sin gt[\sqrt{(n+1)}\sqrt{(n+2)}..\sqrt{(n+m)}]|b_1,a_2,n+m\rangle \quad (8)$$

The state vector representing the two atoms and the cavity for $t>2\tau$ is given by

$$|\psi(t)\rangle_{a-a-f} = \gamma_1|a_1,a_2,n\rangle + \gamma_2|a_1,b_2,n+m\rangle + \gamma_3|b_1,a_2,n+m\rangle + \gamma_4|b_1,b_2,n+2m\rangle \quad (9)$$

Here a_2 and b_2 are the upper and lower states respectively of the second atom. The

$\gamma's$ are given by

$$\gamma_1 = \cos^2 gt\sqrt{(n+1)}\sqrt{(n+2)}....\sqrt{(n+m)}. \quad (10)$$
$$\gamma_2 = \{\cos gt\sqrt{(n+1)}\sqrt{(n+2)}....\sqrt{(n+m)}\}\{-i\sin gt\sqrt{(n+1)}\sqrt{(n+2)}....\sqrt{(n+m)}\}$$
$$\gamma_3 = \{-i\sin gt\sqrt{(n+1)}\sqrt{(n+2)}....\sqrt{(n+m)}\}\{\cos gt\sqrt{(n+m+1)}\sqrt{(n+m+2)}....\sqrt{(n+2m)}\}$$
$$\gamma_4 = \{-i\sin gt\sqrt{(n+1)}\sqrt{(n+2)}....\sqrt{(n+m)}\}\{-i\ \sin gt\sqrt{(n+m+1)}\sqrt{(n+m+2)}....\sqrt{(n+2m)}\}$$

Since the joint state of two atoms emanating from the cavity is not a pure state, the entanglement of the two-atom system can be quantified by the concurrence as proposed by Wooters [11]. This has been widely used to study bipartite entanglement. The concurrence of the system is given by

$$C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\} \quad (11)$$

where λ are the four eigenvalues of the non-Hermitian matrix

$$R = \rho(\sigma_y \otimes \sigma_y)\rho^{\bullet}(\sigma_y \otimes \sigma_y) \quad (12)$$

arranged in a decreasing order, ρ is the (4×4) density matrix of the two-atom system. Entanglement can be quantified by another function, called the entanglement of formation $E_f(\rho)$, monotone of C. It can be defined as

$$E_f(\rho) = h\left[\frac{1+\sqrt{1-C^2(\rho)}}{2}\right] \quad (13)$$

where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$

The effect of field statistics on the atom-atom entanglement has been studied in the Jaynes Cummings Model and Intensity Dependent Jaynes Cummings Model and it has been noticed that the atom-atom entanglement is sensitive to the field statistics [5, 6,13-17]. So, it is not out of place to study this effect in the present work, so we have studied the coherent state which exhibits Poisson distribution.

4.3. ENTANGLEMENT WHEN FIELD IN A COHERENT STATE

Coherent state cavity field:

A coherent state is a minimum uncertainty state standing at the threshold of the classical quantum limit [14]. These states are represented by a single complex number α as follows:

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{14}$$

A Coherent state is an eigenstate of the annihilation operator α

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \tag{15}$$

These states obey a Poissonian distribution function in photon number representation.

$$P(n) = |\langle n\|\alpha\rangle|^2 = \exp[-|\alpha|^2]\frac{\alpha^{2n}}{n!} = \exp[-\bar{n}]\frac{\bar{n}^n}{n!} \tag{16}$$

When we consider the cavity mode with frequency ω to be in an initial coherent state, the complete wavefunction of the model gets reduced to

$$|\psi(t)_{a-a-f}\rangle = \sum_{n=0}^{\infty} C_n[\gamma_1|a_1,a_2,n\rangle + \gamma_2|a_1,b_2,n+m\rangle + \gamma_3|b_1,a_2,n+m\rangle + \gamma_4|b_1,b_2,n+2m\rangle] \tag{17}$$

where, $P_n = |C_n|^2$

Since we are interested in calculating the entanglement of the joint two-atom state after the atoms emerge from the cavity, we consider the reduced density state $\rho(t)_{a-a}$ of the two atoms obtained after taking trace over the field variables.

$$\rho(t)_{a-a} = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} \langle n_1,n_2\|\psi(t)_{a-a-f}\rangle\langle\psi(t)_{a-a-f}|]\|n_1,n_2\rangle$$

We obtain the matrix elements of $\rho_{atom-atom}$ making use of the above expression as:

$$\rho_{11} = \sum_{n=0}^{\infty}|C_n|^2|\gamma_1(n)|^2 \qquad \rho_{12} = \sum_{n=0}^{\infty} C_{n+m}\,\gamma_1(n+m)C_n^*\gamma_2^*(n)$$

$$\rho_{13} = \sum_{n=0}^{\infty} C_{n+m}\,\gamma_1(n+m)C_n^*\gamma_3^*(n) \qquad \rho_{14} = \sum_{n=0}^{\infty} C_{n+2m}\,\gamma_1(n+2m)C_n^*\gamma_4^*(n)$$

$$\rho_{21} = \sum_{n=0}^{\infty} C_n\,\gamma_2(n)C_{n+m}^*\gamma_1^*(n+m) = -\rho_{12} \qquad \rho_{22} = \sum_{n=0}^{\infty}|C_n|^2|\gamma_2(n)|^2$$

$$\rho_{23} = \sum_{n=0}^{\infty}|C_n|^2\,\gamma_2(n)\,\gamma_3^*(n) \qquad \rho_{24} = \sum_{n=0}^{\infty} C_{n+m}\,\gamma_2(n+m)C_n^*\gamma_4^*(n)$$

$$\rho_{31} = \sum_{n=0}^{\infty} C_n\,\gamma_3(n)C_{n+m}^*\gamma_1^*(n+m) = -\rho_{13} \qquad \rho_{32} = \sum_{n=0}^{\infty}|C_n|^2\,\gamma_3(n)\,\gamma_2^*(n) = \rho_{23}$$

$$\rho_{33} = \sum_{n=0}^{\infty}|C_n|^2|\gamma_3(n)|^2 \qquad \rho_{34} = \sum_{n=0}^{\infty} C_{n+m}\,\gamma_3(n+m)C_n^*\gamma_4^*(n)$$

$$\rho_{41} = \sum_{n=0}^{\infty} C_n \ \gamma_4(n) C^*_{n+2m} \gamma^*_1(n+2m) = \rho_{14} \qquad \rho_{42} = \sum_{n=0}^{\infty} C_n \ \gamma_4(n) C^*_{n+m} \gamma^*_2(n+m) = -\rho_{24}$$

$$\rho_{43} = \sum_{n=0}^{\infty} C_n \ \gamma_4(n) C^*_{n+m} \gamma^*_3(n+m) = -\rho_{34} \qquad \rho_{44} = \sum_{n=0}^{\infty} |C_n|^2 |\gamma_4(n)|^2 \tag{18}$$

A detailed analysis of atom-atom entanglement, $E_f(\rho)$ in eq.(13) for arbitrary interaction needs to be evaluated numerically. We compute the entanglement of formation E_f for processes of different orders and plotted versus Rabi angle g_t for different values of n [16].

4. SINGLE MODE TWO PHOTON PROCESS

In the expression for state vector given by eqn. (9), putting for m = 2, we get the single mode two photon Jaynes Cummings model for which the atom – atom entanglement is computed and plotted versus g_t.

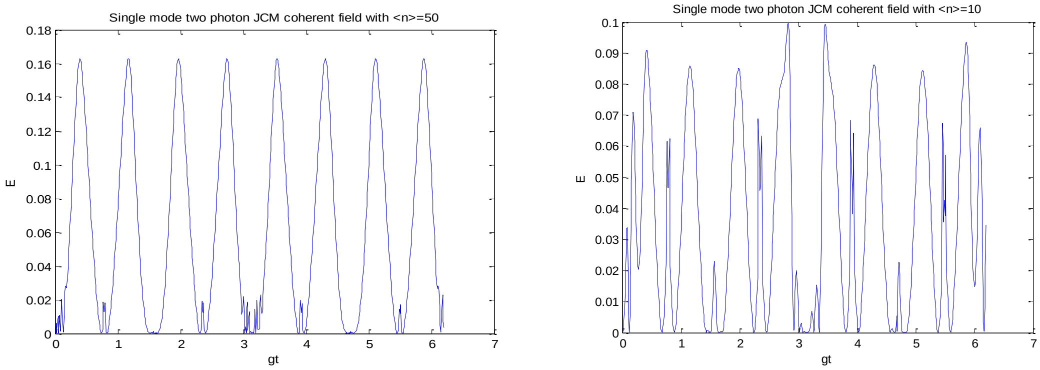

Fig. 1. Evolution of atomic entanglement for both the atoms initially in the excited state and the field in a coherent state (a) $\langle n \rangle = 50$**; (b)** $\langle n \rangle = 10$

5. SINGLE MODE THREE PHOTON PROCESS

The general state vector eqn.(9) gives the single mode three photon JCM when the value of m is taken to be 3. Using the method used above, again the entanglement computed and analyzed graphically versus g_t.

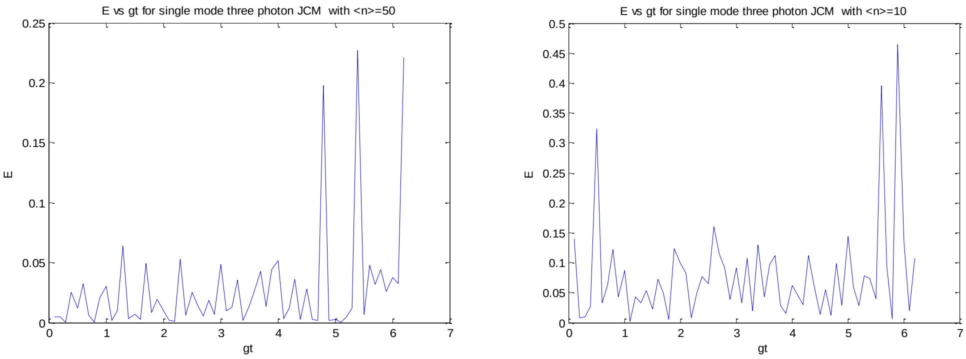

Fig. 2. Evolution of atomic entanglement for both the atoms initially in the excited state and the field in a coherent state (c) $\langle n \rangle = 50$ **; (d)** $\langle n \rangle = 10$

6. SINGLE MODE FOUR PHOTON PROCESS

We have also obtained the single mode four photon using Jaynes Cummings model [15, 16] by taking the value of m to be 4 in eqn. (9). In this case also, the variation of entanglement has been plotted and compared with the other processes.

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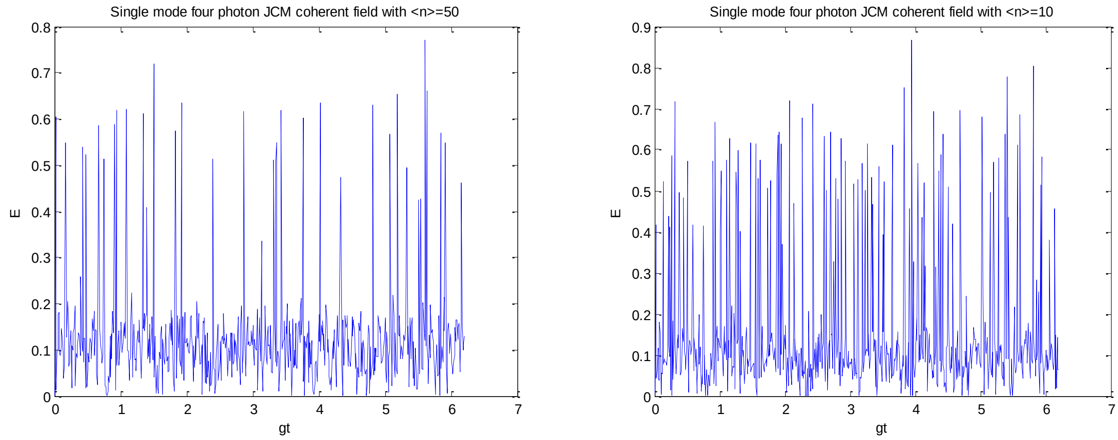

Fig. 3. Evolution of atomic entanglement for both the atoms initially in the excited state and the field in a coherent state (c) $\langle n\rangle = 50$**; (d)** $\langle n\rangle = 10$

7. DISCUSSION AND CONCLUSION

We have plotted the entanglement of formation in the three processes with single mode and different number of photons of the cavity field as a function of Rabi angle g_t. In all the cases, the plots are a series of maxima and minima with respect to $g_t$with a finite value of entanglement even for vacuum field. The variation of entanglement is studied for both low and high values of $\langle n\rangle$showing sharply defined peaks for higher values.

We notice that the two atoms disentangle from each other periodically and then get entangled again. This is a result of superposition of oscillatory behaviour of E_f for each $|n\rangle$. The peak value of E_f decreases with average photon number. $\langle n\rangle = |\alpha|^2$. The entanglement of formation which is a monotone of concurrence ranges from 0 to a maximum value and several intermediary values also. This is due to the fact that C which a function of sine terms may acquire values between 0 and a maximum.

Also, a comparative analysis of single mode two photon, single mode three photon and single mode four photon process reveals that it shows that entanglement increases with the order of interactions.

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